Distributed algorithm for simulating dynamic interactions within a general cyber-physical system

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MIT EECS

MIT Electric Energy Systems Group

Power systems-present and future

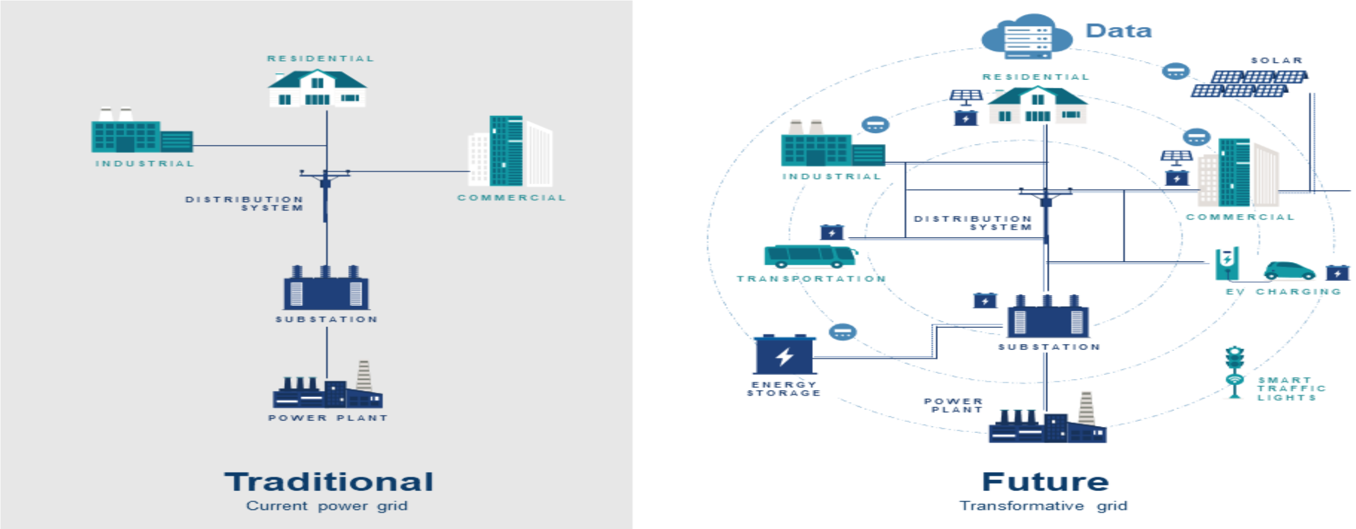

Fig. 1. Past and the future of power systems

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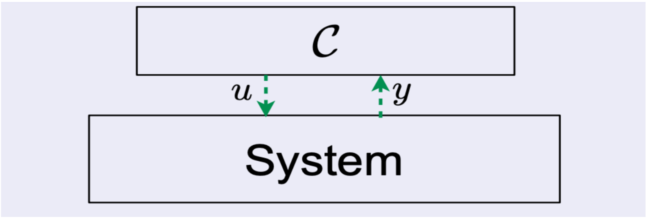
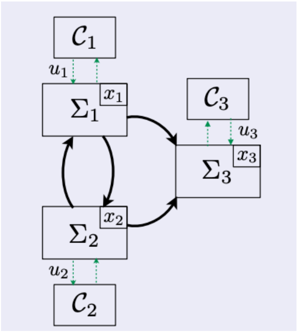
https://www.tva.com/energy/technology-innovation/regional-grid-transformation

State of Art Simulation of Physical Systems

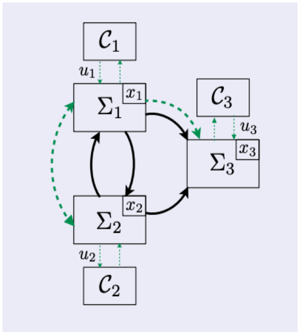
- Different simulation approaches
- Centralized approach pitfalls
  - Scalability issue increases with the number of areas
  - Compatibility issue with different regulation authorities
  - How does one differentiate between numerical and physical issue when something unexpected happens?
- Last but not least, how does one decompose hard-coupled systems, while providing feasibility conditions?



a) centralized



b) decentralized



c) distributed

Fig. 3. Simulation strategies of physical system

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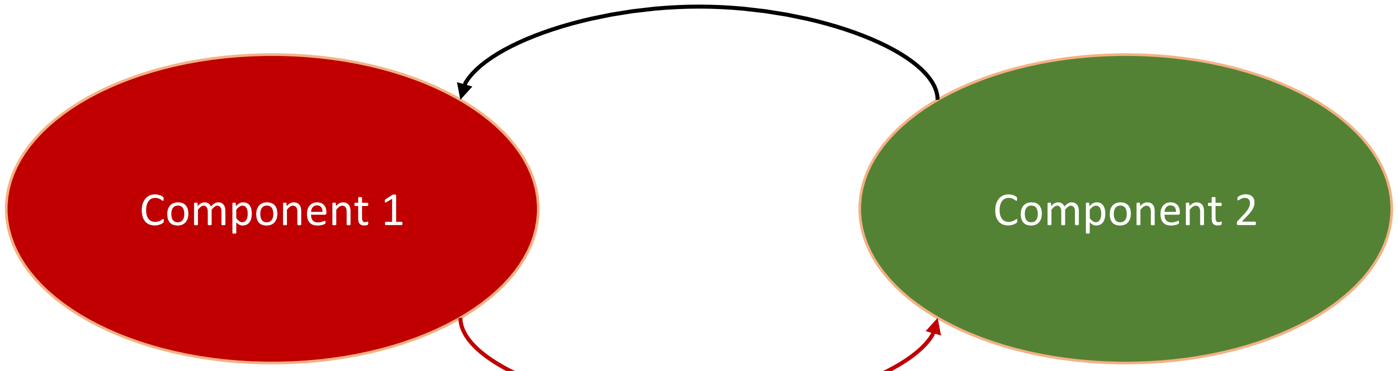
Distributed Simulation high-level Zoom-Out Interaction…

$$\dot{z}_2^{out} = \begin{bmatrix} P_2^{out} \\ \dot{Q}_2^{out} \end{bmatrix}$$



$$\dot{z}_1^{out} = \begin{bmatrix} P_1^{out} \\ \dot{Q}_1^{out} \end{bmatrix}$$

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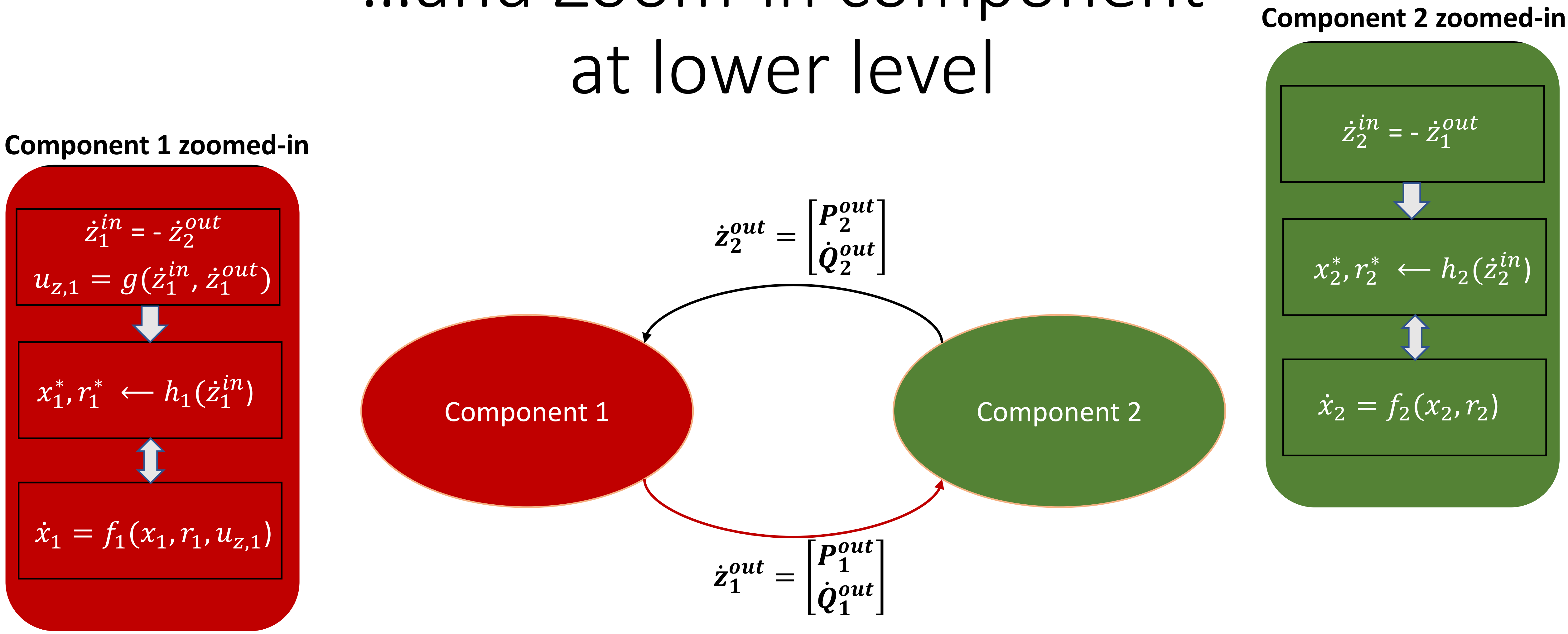
…and Zoom-in component at lower level



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One possible implementation

Summary of algorithm steps

1. Initialize states and port variables of each module
2. Compute dynamics of each module
3. Compute and send intVars
4. Receive and map intVars to conventional space
5. Update value of port variable and repeat step 2.

Algorithm Interactive alignment of components **i** and **j** via P and $\dot{Q}$

Initialize $x_i[0], r_i[0], \epsilon$ and $u_i[0]$ if applicable
$\frac{dr_i}{dt}[0] = 0$
$\frac{dx_i}{dt}[0] = f_i(x_i[0], r_i[0], u_i[0])$
$P_i^{out}[0] = r_i[0]x_i[0]$
$\dot{Q}_i^{out}[0] = -\left(r_i[0]\frac{dx_i}{dt}[0] - x_i[0]\frac{dr_i}{dt}[0]\right)$
for $k = 1, 2, \ldots, N$ **do**
  $P_i^{in}[k] = P_j^{out}[k-1]$
  $\dot{Q}_i^{in}[k] = \dot{Q}_j^{out}[k-1]$
  if k == 1 **then**
    $\frac{dr_i}{dt}[k] = \epsilon$
  else if k == 2 **then**
    $r_i^*[k-1] = \frac{\left(\dot{Q}_i[k]-\dot{Q}_i[k-1]\right)}{\epsilon}$
    $x_i^*[k] = \frac{P_i^{in}[k-1]}{r_i^*[k-1]}$
    $r_i^*[k] = \frac{P_i^{in}[k]}{x_i^*[k]}$
    $\frac{dr_i}{dt}[k] = \frac{r_i^*[k]-r_i[k-1]}{dt}$
  else
    $r_i^*[k] = \frac{P_i^{in}[k]}{x_i[k-1]}$
    $\frac{dr_i}{dt}[k] = \frac{r_i^*[k]-r_i[k-1]}{dt}$
  end if
  $\frac{dx_i}{dt}[k] = f_i(x_i[k-1], r_i[k-1], u_i[k-1])$
  $r_i[k] = r_i[k-1] + \frac{dr_i}{dt}[k]dt$
  $x_i[k] = x_i[k-1] + \frac{dx_i}{dt}[k]dt$
  $P_i^{out}[k] = -r_i[k]x_i[k]$
  $\dot{Q}_i^{out}[k] = -\left(r_i[k]\frac{dx_i}{dt}[k] - x_i[k]\frac{dr_i}{dt}[k]\right)$
end for

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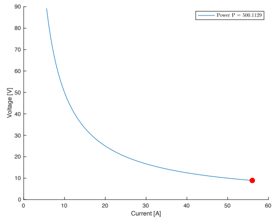
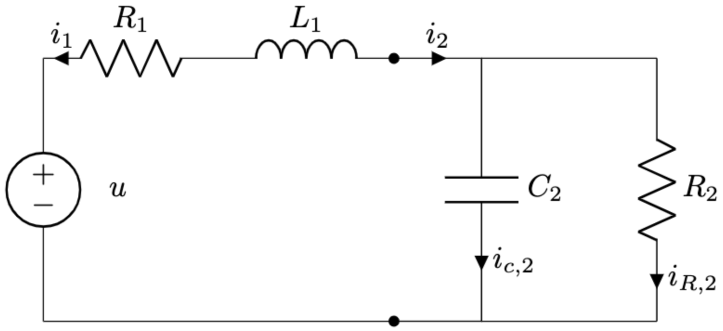
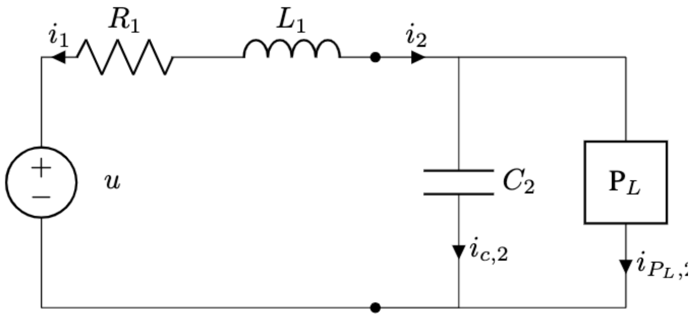
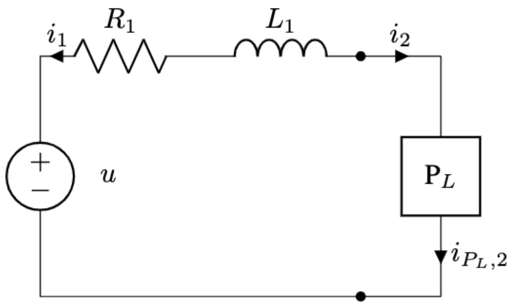
RLC circuit-Simple model of a DC microgrid

- Why does this problem matter?
  - Well…CPL acts as a negative resistance
- How to approach simulation of this problem through the design of control for instability caused by CPLs in a DC microgrid?
- What we propose and we believe is needed?
  - Alignment of not only P at component interfaces, but also $\dot{Q}$ at the higher level (Tellegen's General Theorem-distributed), but also i and v (decentralized) so as to satisfy KCL and KVL





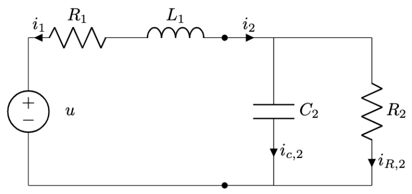


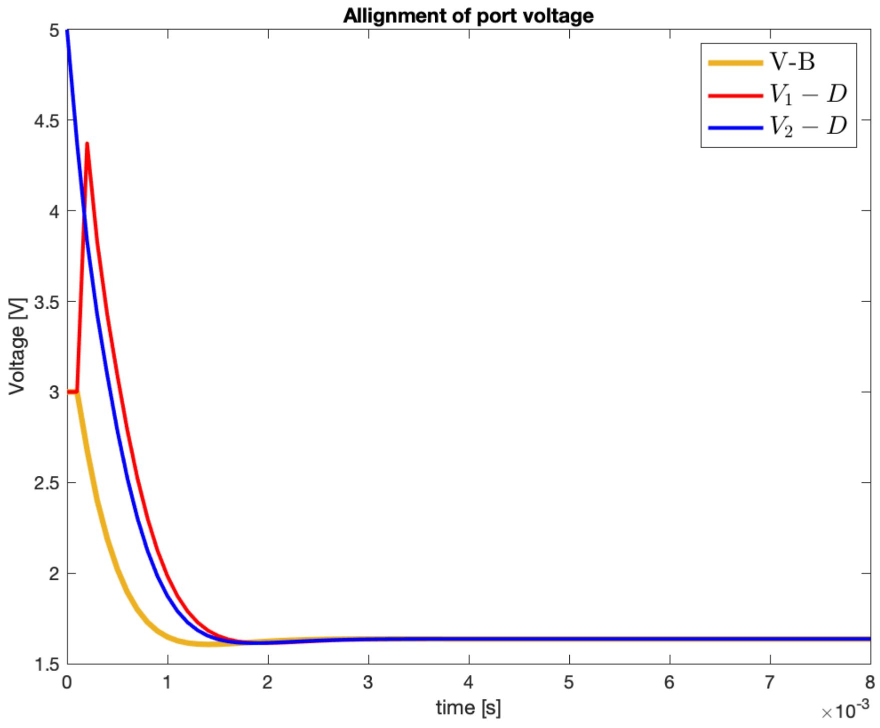


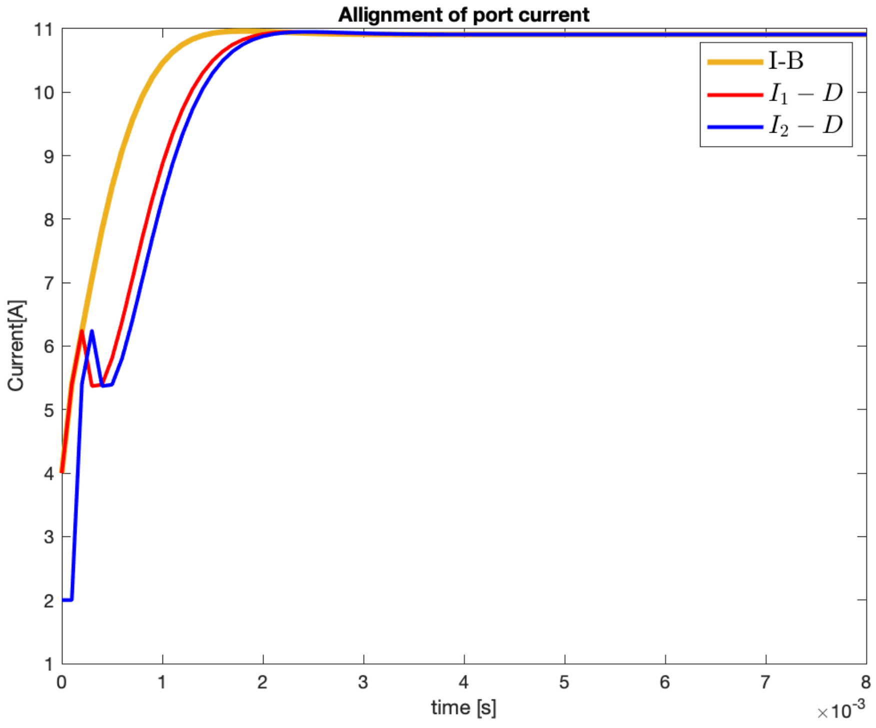
$$i1 = -4\ A, v1 = 3V,, u\ = 6\ V$$
$$R_1\ = \ 0.4\ Ohm, L_1 = 1 * 10^{-4}\ H$$

$$i2\ = 2\ A, v2\ = 5\ V$$
$$R_2 = 0.15\ Ohm, C_2 = 5 * 10^{-3}\ \ F$$

Results







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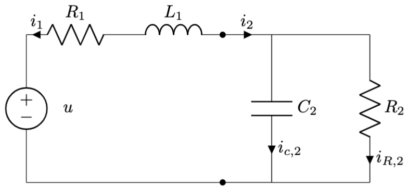

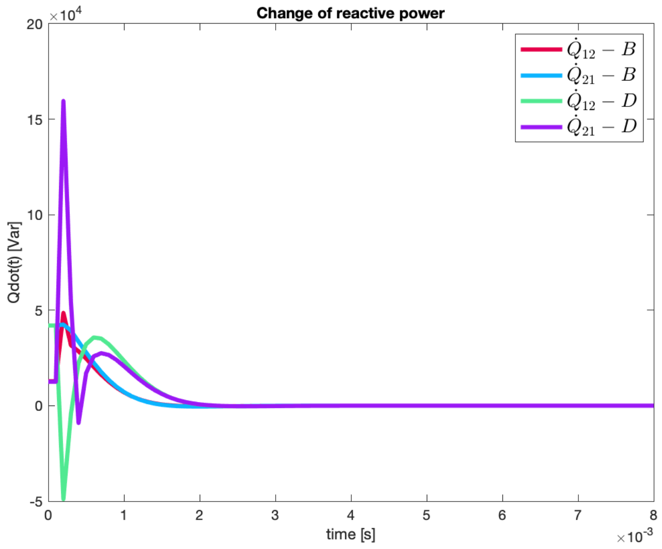

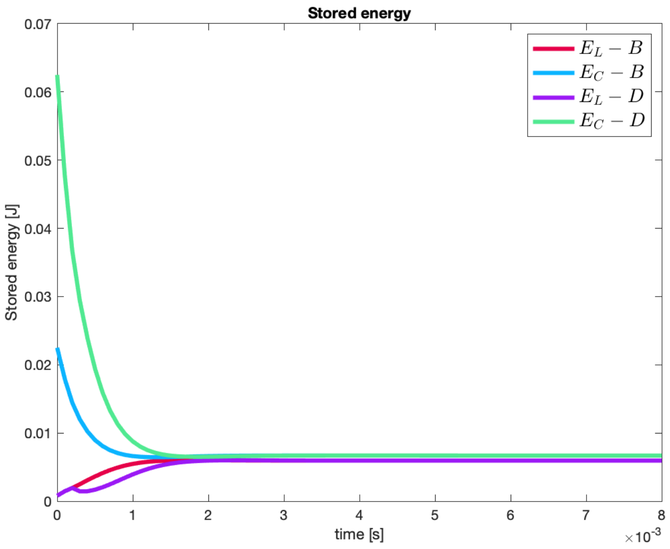

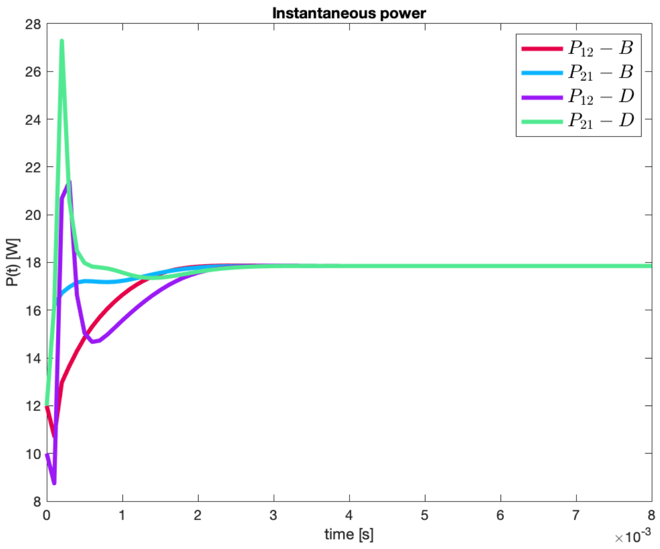

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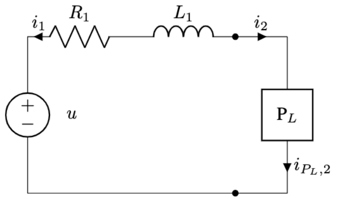
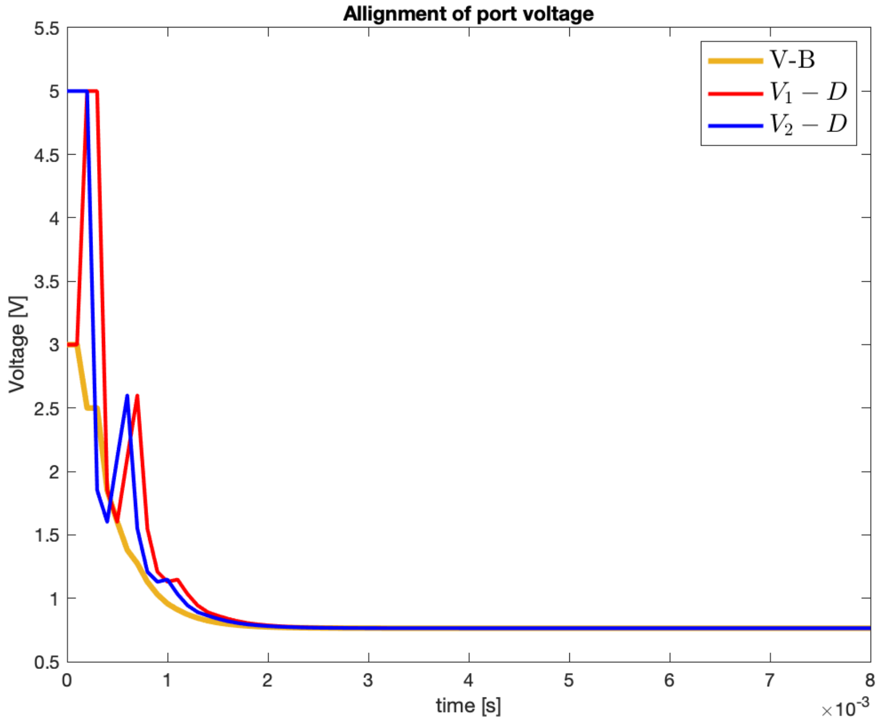
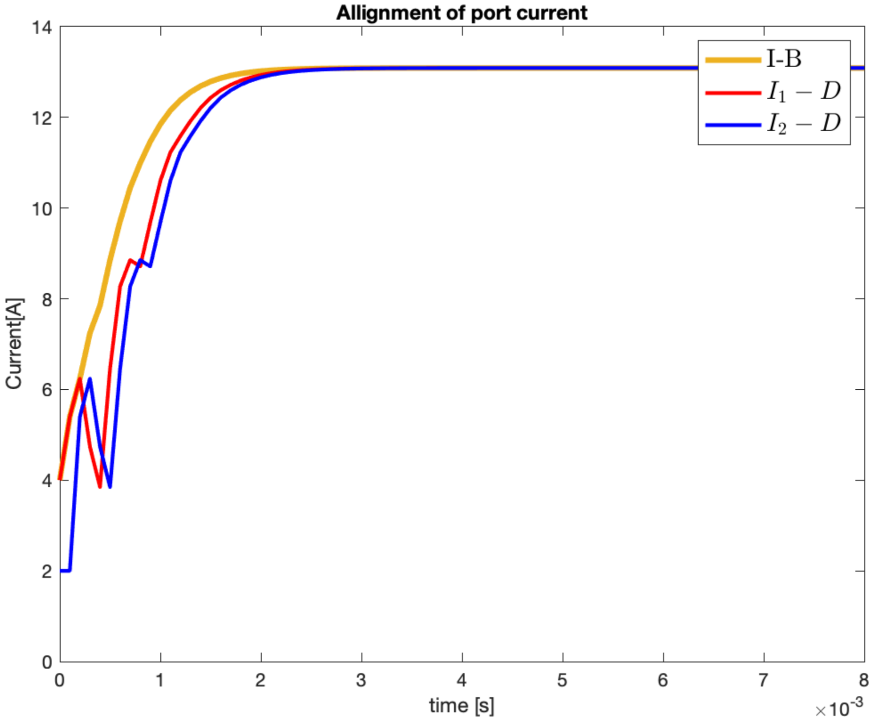
$P_L = 10\ W$

Results







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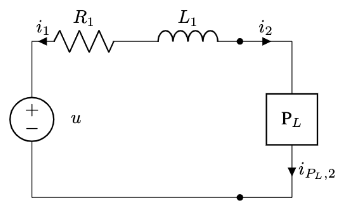

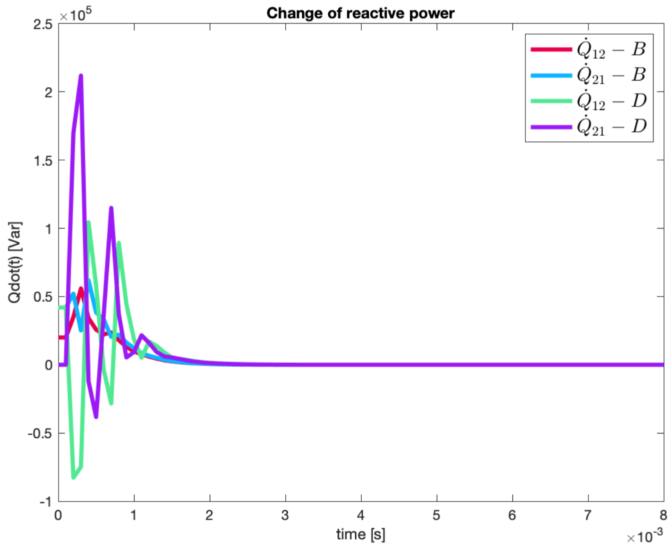

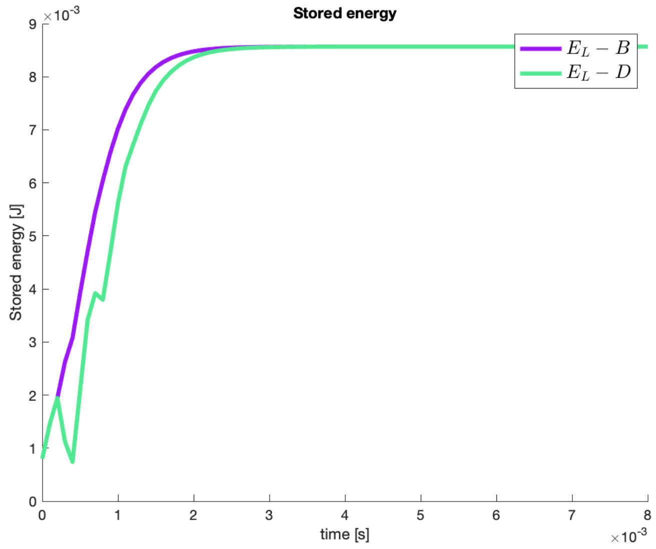

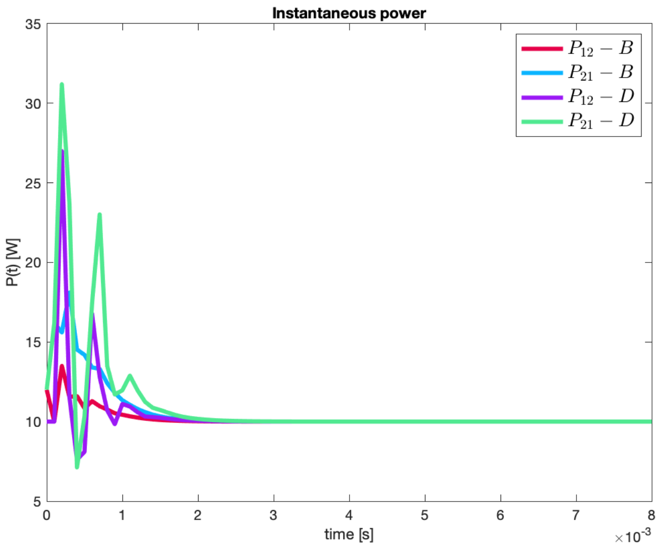

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Thank you!